

# University Center for Mathematical Modeling, Applied Analysis and Computational Mathematics

Semester Seminar, hybrid form (in person and via Zoom),  
New members: K3, Wednesdays, April 24, May 3 and 17, 09:00-10:00  
Remaining members: K8, Wednesday, May 24, 9:00-10:40

## SCHEDULE

Time & Space	Speaker	Title
Wednesdays, K3		New members
19.4. 9:00–10:00	Vojtěch Patočka	Dynamic reorientation of tidally locked bodies: application to Pluto
26.4. 9:00–10:00	Lenka Slavíková	Singular integral operators from different perspectives
3.5. 9:00–10:00	Petr Pelech	Modeling of bacterial cell division
17.5. 9:00–9:30	Jan Blechta	GMRES with I+compact is superlinear. What about A+compact?
17.5. 9:30–10:00	Daniel Campbell	Injectivity in second-gradient Nonlinear Elasticity
Wednesday, May 24, K8		Remaining members: 15+5 minutes
9:00–9:20	Michal Pavelka	Geometric models of superfluid helium-4
9:20–9:40	Ondřej Chrenko	Accreting luminous planets can escape from pressure bumps
9:40–10:00	Petr Šácha	Unravelling climate impacts of atmospheric internal gravity waves
10:00–10:20	Hina Arif	Elasticity of nanostructures
10:20–10:40	Stefano Pozza	A closed-form expression ODE's solution in a ring of distributions and its connection with the matrix algebra

Note: Wednesday, May 10, 2023: Trip to Prokopské údolí

## ABSTRACTS

**Hina Arif: Elasticity of nanostructures.** In the first part of my presentation, I will discuss the stability of nanobeams with defects. For the second part, I will focus on mitochondrial membranes and their fusion and fission processes. Discontinuities like cracks, steps and various boundary and intermediate conditions affect the stability of nanobeams and nanoplates. The sensitivity of critical buckling load and critical stress concerning different geometrical and physical parameters of Euler-Bernoulli nanobeams with defects will be discussed. Eringen's nonlocal theory of elasticity determines critical buckling load for stepped nanobeams subjected to axial loads for different support conditions. An analytical approach has been developed to study the impact of discontinuities and boundary conditions on the critical buckling load and critical stress of nanobeams. Critical buckling loads of stepped nanobeams are defined under the condition that the nanoelements are weakened with stable crack-like defects. Mitochondria are bilayer membrane organelles with a unique structure composed of lipids and proteins. The inner membranes of these organelles contain folds known as cristae, which act as a barrier. Lipid bilayer fusion is characterized by the formation of a "fusion stalk," which is then extended to a local hemifusion connection between two membranes. I'll discuss the unknown stages that follow the formation of the stalk and Hemifusion phase which ultimately lead to the rupture of hemifusion and results in a fusion pore. We employed the Hamm-Kozlov-Helfrich energy model to calculate the entire least energy pathway of membrane fusion from stalk formation to hemifusion and pore formation.

Additionally, I will discuss lipid mixing, an essential component of the fusion process that enables the amalgamation of two distinct lipid bilayers into a continuous membrane. Our collaborators provided us with cryo-electron tomograms of mitochondrial cristae, and we will utilize these structures to investigate the elastic properties of bilayer membranes.

**Jan Blechta: GMRES with I+compact is superlinear. What about A+compact?** It is known for decades that Krylov subspace methods applied to compact perturbations of identity produce superlinear convergence and that the convergence behavior is bounded in terms of the distribution of the singular values of the perturbation. In this talk we will present a generalization of such kind of result. We consider operators that exhibit linear GMRES convergence. Such operators are precisely those that do not contain the origin in the closure of their numerical range. The main result is stability with respect to compact perturbations: If the operator is compactly perturbed, deviation from the linear convergence is controlled in terms of the singular values of the perturbation. We will give this statement a precise quantitative meaning, which allows for arbitrarily large perturbations. We will mention applications of this result and state some open questions. This result has been published as [J. Blechta, SIMAX 2021, <https://doi.org/10.1137/20M1340848>].

**Daniel Campbell: Injectivity in second-gradient Nonlinear Elasticity.** We study injectivity for models of Nonlinear Elasticity that involve the second gradient. We assume that  $\Omega \subset \mathbb{R}^n$  is a domain,  $f \in W^{2,q}(\Omega, \mathbb{R}^n)$  satisfies  $|J_f|^{-a} \in L^1$  and that  $f$  equals a given homeomorphism on  $\partial\Omega$ . Under suitable conditions on  $q$  and  $a$  we show that  $f$  must be a homeomorphism. As a main new tool we find an optimal condition for  $a$  and  $q$  that imply that  $\mathcal{H}^{n-1}(\{J_f = 0\}) = 0$  and hence  $J_f$  cannot change sign. We further specify in dependence of  $q$  and  $a$  the maximal Hausdorff dimension  $d$  of the critical set  $\{J_f = 0\}$ . The sharpness of our conditions for  $d$  is demonstrated by constructing respective counterexamples.

**Ondřej Chrenko: Accreting luminous planets can escape from pressure bumps.** Pressure bumps are radial perturbations in protoplanetary discs that trap drifting dust and pebbles. Several recent studies have argued that pressure bumps might be efficient sites of planetary accretion because if a planetary embryo is formed there, it should be protected by the bump against planetary migration and, at the same time, it can feed from the replenishing reservoir of dust and pebbles. However, none of these works have considered the influence of thermal forces that arise when planets release the accretion heat into the surrounding gas. Using high-resolution 3D hydrodynamic simulations with radiation diffusion, I find that when the thermal forces are accounted for, the planets often become eccentric and they migrate away from the pressure bump. This effect can limit the final masses of planets born within pressure bumps.

**Vojtěch Patočka: Dynamic reorientation of tidally locked bodies: application to Pluto.** Planets and moons reorient in space due to mass redistribution that is associated with various types of internal and external processes. While the equilibrium orientation of a tidally locked body is well understood, much less explored are the dynamics of the reorientation process (or true polar wander, TPW, used here for the motion of either the rotation or the tidal pole). The talk will cover the basic theory and observations, and our numerical solution to the problem will be outlined.

**Michal Pavelka: Geometric models of superfluid helium-4.** At very low temperatures (less than 2.17 K), helium becomes superfluid. For instance, it flows through narrow capillaries with no resistance. Although traditional models describe superfluid helium as a mixture of a normal component and a superfluid component, helium is actually a pure substance, not a mixture, that exhibits two motions. In order to describe the two motions of superfluid helium, we use geometric mechanics, that connects classical continuum mechanics with the underlying quantum mechanics. Another aspect of superfluid helium is the presence of quantum vortices, topological defects of the phase of the wave function. It remains unclear, however, how to properly describe flows that contain quantum vortices.

**Petr Pelech: Modeling of bacterial cell division.** To divide, bacteria must remodel their wall at the division site. However, how the cell overcomes the internal turgor pressure to reduce its diameter remains unclear. It is debated, whether remodeling alone can drive membrane constriction or if an additional constrictive force is required.

**Stefano Pozza: A closed-form expression ODE's solution in a ring of distributions and its connection with the matrix algebra.** A new expression for solving homogeneous linear ODEs based on a generalization of the Volterra composition was recently introduced. In this work, we extend such an expression, showing that it corresponds to inverting an infinite matrix. This is done by studying a particular subring and connecting it with a subalgebra of infinite matrices.

**Lenka Slavíková: Singular integral operators from different perspectives.** In this talk, I will introduce my research, which involves (multi)-linear singular integral operators and their applications in ergodic theory, as well as somewhat related topics featuring the specific singular integral operator called fractional Laplacian and associated function spaces.

**Petr Šácha: Unravelling climate impacts of atmospheric internal gravity waves.** Unresolved processes in climate models present a major source of uncertainty in future climate projections. In this talk we will show the first results and introduce a newly founded five year project, which aims to re-examine the climate impacts of atmospheric internal gravity waves (GWs) using GW resolving simulations and to translate this knowledge to the development of modified GW parameterizations in climate models. Within the project, we will employ state-of-the-science high-resolution atmospheric datasets and theoretical methods for GW detection and wave-mean flow interaction to revisit and advance our understanding of GW effects on atmospheric dynamics, composition and coupling across atmospheric layers.

GWs exist on a variety of scales, but typically a significant portion of the GW spectrum remains unresolved in global weather prediction or climate models and the GW impacts need to be parameterized. Our knowledge on GW impacts ranging from regionality of precipitation to the evolution of the ozone layer has been so-far based on their predominantly parameterized effects. Analyzing the resolved GW effects will improve our understanding on the forcing of selected atmospheric phenomena, but will also put additional constraints on the current GW parameterizations by showing to what extent their effects (and our current understanding) are artificial. This will help us to modify GW parameterization schemes with an ultimate goal of alleviating the uncertainty in future climate projections.