University Center for Mathematical Modeling, Applied Analysis and Computational Mathematics

Semester Seminar, **virtual form** (via Zoom),
Wednesday December 1, 2021, 8:25-12:10

Schedule

Time	Speaker	Title
Morning I		Zoom; Chair: Miroslav Bulíček
8:25	*Opening*	
8:30	Sebastian Schwarzacher	Some recent progress in the analysis of fluid-structure interactions
8:50	Malte Kampschulte	Variational approach to fluid structure interaction - the compressible case
9:10	Marek Cúth	Projections induced by group actions
9:30	*Short break*	10 min
Morning II		Zoom; Chair: Vít Průša
9:40	Ondřej Chrenko	The inner rim of protoplanetary disks perturbed by an embedded planet
10:00	Josef Hanuš	Physical Properties of Near-Earth Asteroids
10:20	Michal Pavelka	Hamiltonian mechanics of superfluid helium-4
10:40	*Break*	20 min
Morning III		Zoom; Chair: Jaroslav Hron
11:00	Gianmarco Sperone	Further remarks on radial symmetry and monotonicity for solutions of semilinear higher order elliptic equations
11:20	Scott Congreve	An Iterative Discontinuous Galerkin Finite Element Method for Strongly Monotone Quasi-Linear Elliptic PDEs
11:40	Stefano Pozza	The $*$-product and its numerical approximation
12:00	*Closing discussion*	10 min

Abstracts

Scott Congreve: An Iterative Discontinuous Galerkin Finite Element Method for Strongly Monotone Quasi-Linear Elliptic PDEs. In previous work [C. and Wihler, 2017] we investigated the use of fixed point iterations to solve the Galerkin approximation of strictly monotone problems. As opposed to Newton's method, which requires information from the previous iteration in order to *linearize* the iteration matrix (and thereby to recompute it) in each step, the alternative method used exploited the monotonicity properties of the problem, and only needs the iteration matrix calculated once for all iterations of the fixed point method.

We extend this work to an an interior penalty discontinuous Galerkin finite element formulation of strongly monotone quasi-linear elliptic partial differential equations. We outline the *a priori* and *a posteriori* error estimates for iteratively obtained solutions, and show both theoretically as well as numerically how the number of iterations of the fixed-point method can be restricted in dependence of the mesh size, or of the polynomial degree, to obtain optimal convergence. Furthermore, we demonstrate how the *a posteriori* error analysis can be used to minimise the number of iterations on a sequence of adaptively refined finite element spaces.

Marek Cúth: Projections induced by group actions. Given a compact group acting on a metric space by isometries, we show there exists a canonical linear projection on the Banach space of Lipschitz function associated to this action. We study those new projections and generalize this construction even to the setting which works for any Banach space. The talk is based on a joint preprint with M. Doucha.

Ondřej Chrenko: The inner rim of protoplanetary disks perturbed by an embedded planet. The inner rim of protoplanetary disks is thought to be important for (i) planet formation (because it works as a barrier for migrating planets and thus it naturally explains the existence of numerous close-in exoplanets) and (ii) disk observations (because each disk rim is heavily irradiated by its host star). I model the inner rim using an iterative method that combines hydrostatic equations for the density distribution of gas and time-dependent energy equations for gas and radiation. The stellar irradiation is treated using a simple ray tracing method and the thermal emission is treated using the flux-limited diffusion approximation. The aim is to study how the gravity of an embedded planet changes the structure of the inner rim. My results suggest that planets of relatively low masses (e.g. 10-20 Earth masses) open gaps along their orbits and thus they split the irradiated surface of the inner rim in two regions. These two regions should appear as two distinct emission rings, which I plan to demonstrate in near future by calculating synthetic images with a Monte-Carlo radiative transfer code.

Josef Hanuš: Physical Properties of Near-Earth Asteroids. The recent large number of new optical measurements from various surveys allowed to significantly increase the number of asteroids with known physical properties to more than 3,000. In particular, these properties include rotation states, i.e., sidereal rotation period and the orientation of the spin axis, and convex 3D shape models. Those are routinely derived from disk-integrated optical photometry by the convex inversion method.

Only photometric datasets that sufficiently sample various observing geometries lead to the unique determination of physical properties. It turned out that main-belt asteroids often have such datasets, so the majority of successful shape solutions belong to this category of asteroids. Obtaining good photometry for near-Earth asteroids (NEA) is challenging due to their smaller apparent brightness, larger proper motion on the sky, or less convenient observing opportunities (e.g., closer to the Sun, short observing windows, long intervals between possible observations). Therefore, shape and rotation state properties have been so far derived for only a few tens of NEAs. In our work, we analyze photometric datasets that we gathered from various sources (such as Gaia DR2, ATLAS, ASAS-SN) in an attempt to at least double the number of shape solutions for NEAs.

Malte Kampschulte: Variational approach to fluid-structure interaction - the compressible case. The aim of this talk is to present a recent existence result (obtained with D.Breit and S.Schwarzacher) for a dynamic FSI problem involving a bulk viscoelastic solid and a compressible fluid. The result is obtained using variational techniques similar to those previously presented for the related incompressible problem. The aim of this talk is thus to instead focus more on the underlying modelling aspects. In particular the variational approach allows us to shift the importance from the density-pressure relationship to the density-dependent internal energy as the primary object in the proof. This hints at a great potential for future extensions.

Michal Pavelka: Hamiltonian mechanics of superfluid helium-4. Helium-4 becomes superfluid at very low temperatures, where it exhibits peculiar behavior, caused by the quantum properties of the fluid. For instance, if kept in an open vessel, it climbs up the walls and eventually overflows. How to describe such macroscopic behavior of superfluid helium-4 theoretically? A common approach are the two-fluid models, where helium-4 is split into a non-viscous superfluid component and a viscous normal component. But helium-4 does not contain two components in reality, which makes the two-fluid models questionable. Moreover, there are several versions of the two-fluid models. Hamiltonian mechanics, which follows from quantum mechanics by taking the quasiclassical limit, allows for modeling of superfluid helium-4 without considering two fluids and it shows relations between the various two-fluid models.

Sebastian Schwarzacher: Some recent progress in the analysis of fluid-structure interactions. I will give some summary of the work performed in the last three years by the working group of fluid-structure interactions.

Stefano Pozza: The $*$-product and its numerical approximation. The time-ordered exponential (TOE) is the solution of a system of coupled first-order linear differential equations with generally non-constant coefficients. Despite being at the heart of many problems, the TOE remains elusively difficult to evaluate. Two new approaches for TOE approximation have been introduced in recent years: the path-sum and the *-Lanczos methods. Both of them rely on a convolution-like product and its inverse. An accurate approximation of such a product is the key to the efficient numerical computation of TOEs. We will present some preliminary results for the product approximation based on quadrature formulas and orthogonal function approximation.

Gianmarco Sperone: Further remarks on radial symmetry and monotonicity for solutions of semilinear higher order elliptic equations. Fifty years after the pubblication of Serrin's pioneering paper on symmetry problems arising in potential theory, we consider semilinear polyharmonic equations under Dirichlet boundary conditions in the unit ball of $\mathbb{R}^n$. After discussing radial properties (symmetry and monotonicity) of positive solutions of such equations, we show that, in conformal dimensions $n = 2m$ with $m \geq 1$, the associated Green function satisfies an elegant reflection property involving the inversion in the sphere and the Kelvin transform. This then yields an alternative formula for computing the partial derivatives of the solutions of the polyharmonic problems considered. We also revise a counterexample by Sweers where radial monotonicity fails: by appropriately modifying the source f we numerically obtain radially symmetric and strictly decreasing solutions of the biharmonic equation in the unit ball of $\mathbb{R}^4$ under Dirichlet boundary conditions. This is a joint work with Filippo Gazzola (Politecnico di Milano).