Paris, July 14, 2013

Post-doctoral position
INRIA Paris-Rocquencourt

Project-teams: Alpines & Pomdapi

Subject: Integration of a posteriori error estimates and of stopping criteria for linear and nonlinear solvers into FreeFem++

Description of the project-teams

Pomdapi

The project-team Pomdapi is concerned with the construction and analysis of simulation tools for mathematical modelling, with applications in particular to environmental and energy problems. These tools include numerical approximation schemes for partial differential equations, nonlinear solvers, numerical techniques in optimization and complementarity problems, a posteriori error estimates, and adaptivity. We are equally interested in reliable and correct programming methods for the implementation of these tools. In particular, concerning programming models, we are interested in high-performance computing, with the development of new algorithms such as space-time domain decomposition, parallel implementation for large scale computations, reliable and correct programming for scientific computing, and formal proofs of correctness for numerical programs. Our research in numerical analysis focuses on guaranteed and robust a posteriori error estimates. These are fully computable quantities allowing to tightly bound the error in a numerical approximation of a partial differential equation. We are also developing fully adaptive strategies, combining adaptive stopping criteria with adaptive space and time mesh refinement.

Alpines

Alpines is a joint research group between INRIA and Jacques-Louis Lions Laboratory, Pierre and Marie Curie University, which focuses on high performance scientific computing. The topics studied in the group are mesh generation for parallel computation, linear solvers based on domain decomposition methods and incomplete factorizations, and computational kernels for sparse and dense linear algebra. The developed numerical algorithms are regularly integrated into FreeFem++ package and are validated on challenging numerical simulations in collaboration with our academic and industrial partners. The linear solvers and computational kernels include multilevel domain decomposition methods with adaptive coarse spaces, multilevel direction preserving factorizations, and communication avoiding algorithms.

Description of the project

Introduction

A large number of environmental and physical phenomena is described by partial differential equations. Unfortunately, in most of the cases, it is not possible to find the exact solutions of these equations. Then *numerical methods* are used as *simulation* tools. Two extremely important questions are:

1. How large is the overall error between the exact and approximate solutions?
2. Where is the error localized?

It is the theory of *a posteriori error estimation* which allows to give answers to these questions.

In general, an optimal numerical algorithm has to be adaptive and ensure that:

3. A precision, given before the simulation start, is attained at the end of the simulation.
4. As small as possible amount of computational work is needed.

A numerical resolution is a complex procedure which starts by the specification of the problem, the choice and implementation of the numerical method, the choice and implementation of the linearization method (nonlinear solver), and finally the choice and implementation of the method of solution of the associated linear systems (linear solver). The scientific calculation code FreeFem++ [4] is an excellent example of a software tool encompassing the totality of such a procedure.

Project

The goal of this project is the integration of the most recent advances of the theory of a posteriori error estimation and of adaptive algorithms into the FreeFem++ code. More precisely, in the first time, we plan to implement the *unified framework* of the *a posteriori estimates* (see for instance [1,3]) for a large spectrum of different problems and numerical methods of FreeFem++. This will allow us to give the answers to the questions 1.–2. above. In the second time, which represents the main novelty of this project, we plan to implement the a posteriori *stopping criteria* for the *linear* and *nonlinear* solvers of FreeFem++. The goal here is to satisfy the properties 3.–4. above and more precisely to obtain substantial economies (typically of one order of magnitude) of the number of total linear solver iterations and, consequently, of the calculation time. We will follow the theory developed in [2] which allows to identify the different error components such as the linearization error, the algebraic error of the solution of linear systems, and the discretization error.

References

[1] Ern, A., Vohralík, M. A posteriori error estimation based on potential and flux reconstruction for the heat equation. *SIAM J. Numer. Anal.* **48** (1) 2010, 198–223.

[2] Ern, A., Vohralík, M. Adaptive inexact Newton methods with a posteriori stopping criteria for nonlinear diffusion PDEs. *SIAM J. Sci. Comput.* **35** (4) 2013, A1761–A1791.

[3] Hannukainen, A., Stenberg, R., Vohralík, M. A unified framework for a posteriori error estimation for the Stokes problem. *Numer. Math.* **122** (4) 2012, 725–769.

[4] Hecht, F., Pironneau, O., Morice, J., Le Hyaric, A., Ohtsuka, K., FreeFem++. Tech. rep., Laboratoire Jacques-Louis Lions, Université Pierre et Marie Curie, Paris, http://www.freefem.org/ff++, 2012.

Candidate profile

Upon taking the position, the candidate needs to have defended the Ph.D. thesis. She or he should have solid knowledge of model partial differential equations (the Laplace equation, the advection–diffusion–reaction equation, the heat equation, the Stokes equation, linear elasticity equations, the Navier–Stokes equations), of basic numerical methods (conforming finite elements, nonconforming finite elements, discontinuous Galerkin methods, mixed finite elements, finite volumes), of nonlinear solvers (Newton method, Pickard method), and of linear solvers (conjugate gradients, BiCGStab, GMRes, multigrid). Moreover, she or he should be strong in C++ programming and should have at least some experience with the FreeFem++ code.

Practicalities

Timing: 16 months, starting autumn 2013.
Location: INRIA Paris-Rocquencourt http://www.inria.fr/en/centre/paris-rocquencourt/ and Jacques-Louis Lions Laboratory, Pierre and Marie Curie University https://www.ljll.math.upmc.fr/en/index.html.
Monthly gross salary: cca 2 600 euros.
The knowledge of French language is welcome but not compulsory.

Application

To apply, send your CV highlighting your background in numerical analysis, scientific computing, and programming, a short motivation letter, a list of your publications, and recommendation letter(s) to Fréderic Hecht (Alpines) frederic.hecht@upmc.fr and Martin Vohralík (Pomdapi) martin.vohralik@inria.fr.