# University Center for Mathematical Modeling, Applied Analysis and Computational Mathematics

Semester Seminar, 20th May 2013, 8:30–13:15, Room K4

## SCHEDULE

| Time | Speaker | Title |
|---|---|---|
| 8:30 | | Opening |
| 8:35 | Miloslav Vlasák | A posteriori error estimates for evolution problems |
| 8:50 | Miroslav Bulíček | Existence analysis for a model describing flow of an incompressible chemically reacting non-Newtonian fluid with power law exponent depending on concentration |
| 9:05 | Václav Kučera | Constructing better numerical methods using reconstruction operators |
| 9:20 | Václav Vlasák | Some results on monotone metric spaces |
| 9:35 | | Discussion |
| 9:45 | | Coffee break |
| 10:05 | Tomáš Bárta | Is every ODE a gradient system? |
| 10:20 | Ondřej Souček | Past glaciation on Mars – a modeling study |
| 10:35 | Karel Tůma | Simulation of viscoelastic Oldroyd-B like model in deforming domains |
| 10:50 | Vít Průša | Fidelity of the estimation of the deformation gradient from the motion of markers placed on a body that is subject to an inhomogeneous deformation field |
| 11:05 | | Discussion |
| 11:15 | | Coffee break |
| 11:35 | Iveta Hnětynková | A new approach to iterative denoising and regularization of inverse problems |
| 11:50 | Ondřej Kurka | Banach–Mazur distance to the cube |
| 12:05 | Martin Lanzendörfer | On some problems arising in hydrodynamic lubrication: questions both old and new |
| 12:20 | Petr Honzík | Dimension of images of subspaces under Sobolev mappings |
| 12:35 | | Discussion |
| 12:45 | | Concluding remarks |

## ABSTRACTS

**Tomáš Bárta, Is every ODE a gradient system?** We show that every autonomous ordinary differential equation with strict Lyapunov function is a gradient systems and some consequences of this fact.

**Miroslav Bulíček, Existence analysis for a model describing flow of an incompressible chemically reacting non-Newtonian fluid with power law exponent depending on concentration.** We consider a system of PDE's describing steady motion of an incompressible chemically reacting non-Newtonian fluid. The main difficulty consists in the fact that the power-law index is a priori unknown and depends on the chemical concentration – one of the unknowns. We introduce a new method – a generalization of the Lipschitz approximation method – and we show the existence of a weak solution provided that we are able to show the Hölder continuity of the power-law exponent. Since the equation for the chemical concentration and consequently also for the exponent is the convection-diffusion equation, the restriction on the Hölder continuity then reduces to the restriction os sufficient integrability of the velocity, which is finally guaranteed by the minimal value of the power-law index, namely $\frac{d}{2}$.

**Iveta Hnětynková, A new approach to iterative denoising and regularization of inverse problems.** Consider a discretized linear inverse problem $Ax \approx b$ with a matrix $A$ representing a model (smoothing operator) and a right-hand side $b$ representing an observation vector. Here the singular values of $A$ decay gradually and $A$ is usually highly ill-conditioned. Moreover, $b$ is often contaminated by *noise* with unknown noise level. The least squares solution of the problem is in most cases dominated by amplified noise. Therefore it is necessary to use *regularization methods* for finding reasonable numerical approximations to the solution.

In I. Hnětynková, M. Plešinger, Z. Strakoš: The regularizing effect of the Golub-Kahan iterative bidiagonalization and revealing the noise level in the data, BIT **49**, pp. 669–696 (2009) regularizing properties of the Golub–Kahan iterative bidiagonalization (GK) has been analyzed. It has been shown how to *estimate the unknown noise level* in the data and also the *noise vector* at a negligible cost directly from the information available during GK. The results apply to white as well as colored noise. We will investigate, how the obtained estimates can further be used in *denoising* and *regularization* of inverse problems.

**Petr Honzík, Dimension of images of subspaces under Sobolev mappings.** Let $m < \alpha < p \leq n$ and let $f \in W^{1,p}(\mathbb{R}^n, \mathbb{R}^k)$ be $p$-quasicontinuous. We find an optimal value of $\beta(n, m, p, \alpha)$ such that for $\mathcal{H}^\beta$ a.e. $y \in (0,1)^{n-m}$ the Hausdorff dimension of $f((0,1)^m \times \{y\})$ is at most $\alpha$. We construct an example to show that the value of the optimal $\beta$ does not increase once $p$ goes below the critical case $p < \alpha$.

**Václav Kučera, Constructing better numerical methods using reconstruction operators.** So-called reconstruction operators are a simple tool often used in the finite volume community, to increase the rate of convergence and accuracy of the classical finite volume method. They are used to reconstruct higher order piecewise polynomial approximations from the piecewise constant approximate solutions. Although not much theory exists, heuristic arguments and numerical experiments show that such procedures indeed improve the quantitative and qualitative accuracy of the first order schemes. We will show how these operators can be successfully applied in other methods, namely the discontinuous Galerkin finite element method.

**Ondřej Kurka, Banach–Mazur distance to the cube.** Known lower and upper bounds on the greatest possible Banach–Mazur distance of a symmetric convex body in an $n$-dimensional space to the $n$-dimensional cube will be discussed.

**Martin Lanzendörfer, On some problems arising in hydrodynamic lubrication: questions both old and new.** We will recall the elements of hydrodynamic lubrication (discussed on the last MathMAC annual seminar) and survey some classic issues (like the reduction of the space dimensions) together with issues raised only recently (non-newtonian lubricant flow modelling and simulation).

**Vít Průša, Fidelity of the estimation of the deformation gradient from the motion of markers placed on a body that is subject to an inhomogeneous deformation field.** Practically all experimental measurements related to the mechanical response of non-linear bodies are concerned with inhomogeneous deformations, though in many experiments much effort is taken to enforce homogeneous deformation fields. However, in experiments that are carried out *in vivo* one cannot control the nature of the deformation. This makes the measurement of the deformation gradient and/or its invariants—which are the key quantities in determining the response of the material—problematic. In such setting, the deformation gradient is usually estimated by tracking positions of a finite number of markers placed in the body.

In our study we are interested in a quantitative description of the difference between the true gradient and its estimate obtained by tracking the markers, that is in the quantitative description of the induced error due to the standard data reduction procedure. We derive a rigorous upper bound on the error and we illustrate the results by studying a practically interesting model problem. We show that different choices of the tracked markers can lead to substantially different estimates of the deformation gradient and its invariants. It is alarming that even qualitative features of the material under consideration such as the incompressibility of the body can be evaluated differently with different choices of the tracked markers. We also demonstrate that the derived error estimate can be used as a tool for choosing the appropriate marker set that leads to the deformation gradient estimate with the least guaranteed error. Finally, we discuss the possibility of exploiting a data reduction procedure based on polynomial approximation and its drawbacks/advantages with respect to the standard data reduction procedure.

**Ondřej Souček, Past glaciation on Mars – a modeling study.** We present numerical simulations of past (3Ga) glaciation of the Isidis Planitia region on Mars. Recently available geological evidence of surface topographical structures supports the scenario of massive past glaciation in the region. We attempt to validate this hypothesis by identifying suitable climatic input conditions (surface temperatures, net surface mass accumulation) and geothermal heat flux, consistent with other modeling studies and the present-day knowledge of geological history of Mars. Our results confirm the possibility of large-scale ice-sheet formation in the region, and suggest a possible interpretation of the surface topographical features as remnants of evolving basal thermal sliding instability.

**Karel Tůma, Simulation of viscoelastic Oldroyd-B like model in deforming domains.** We present a new non-linear thermodynamically compatible rate type fluid model that can be linearized to the standard Oldroyd-B model. This new model is used for simulation of problems in deforming domains using arbitrary Lagrangian-Eulerian method transforming the moving domain to the fixed computational domain. The domain is discretized by regular quadrilaterals. Finite element method with fully coupled monolithic solver is used in numerical computation. Obtained set of non-linear equations is solved by Newton method and the set of linear equations is solved by the direct solver. Pressure is approximated by piecewise discontinuous linear elements and velocity and the part of the stress tensor is approximated by piecewise bi-quadratic continuous elements. In particular we simulate a flow of Newtonian fluid in a viscoelastic deforming channel.

**Miloslav Vlasák, A posteriori error estimates for evolution problems.** We will consider a simple heat equation. This problem will be discretized by several types of methods covering usual methods like Runge–Kutta methods in time and conforming finite elements or discontinuous Galerkin method in space. The known techniques of a posteriori error estimates will be applied and discussed.

**Václav Vlasák, Some results on monotone metric spaces.** Some new results on the recent topic of monotone metric spaces. We proved, e.g., that every 1-monotone metric space in $\mathbb{R}^d$ has finite 1-dimensional Hausdorff measure. As a consequence we obtained that each continuous bounded curve has a finite length if and only if it can be written as a finite sum of 1-monotone continuous bounded curves.