Nonconvolutionary Integrodifferential Equations

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Outline

Jeffrey's fluid (1D model)

$$v_t(t,x) = \sigma_x(t,x) + f(t,x), \quad t \in (0,T),\ x \in (a,b)$$

v velocity, σ stress, f external force.

Constitutive relation (Jeffrey's model)

$$\sigma + \lambda_1 \sigma_t = \varepsilon + \lambda_2 \varepsilon_t$$

($\varepsilon := v_x$ velocity gradient – symmetric part)

$$\sigma(t,x) = \mu_0 \varepsilon(t,x) + \mu_1 \int_0^t e^{-a(t-s)} \varepsilon(s,x) \mathrm{d}s$$

$$v_t(t,x) = \mu_0 v_{xx}(t,x) + \mu_1 \int_0^t e^{-a(t-s)} v_{xx}(s,x) \mathrm{d}s + f(t,x)$$

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Jeffrey's fluid with chemical reactions (1D)

$$v_t = \sigma_x + f.$$

Constitutive relation

$$\sigma(t) = \mu_0 \varepsilon(t) + \mu_1 \int_0^t e^{-a(c(t))(t-s)} \varepsilon(s) \mathrm{d}s$$

$c(t, x)$ concentration, Rajagopal & Wineman.

$$v_t(t) = \mu_0 v_{xx}(t) + \mu_1 \int_0^t (e^{-a(c(t,x))(t-s)} v_x(s))_x \mathrm{d}s + f(t)$$

$$c_t(t) = (\mu(v_x) c_x)_x$$

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Jeffrey's model (3D)

$$v_t + v \cdot \nabla v = \operatorname{div} \sigma - \nabla \pi + f.$$

Constitutive relation

$$\sigma + \lambda_1 \sigma_t = 2\eta (\varepsilon + \lambda_2 \varepsilon_t)$$

(Vorotnikov & Zvyagin, Zvyagin & Dmitrienko)

$$\sigma(t,x) = \mu_0 \varepsilon(t,x) + \mu_1 \int_0^t e^{-a(t-s)} \varepsilon(s, z(s;t,x)) \mathrm{d}s,$$

where $z(s;t,x) = x + \int_t^s v(r, z(r;t,x) \mathrm{d}r$.

$$\begin{aligned} \operatorname{div} v &= 0 \\ v_t + v \cdot \nabla v &= \mu_0 \Delta v + \mu_1 \int_0^t e^{-a(t-s)} \Delta v(s, z(s;t,x)) \mathrm{d}s - \nabla \pi + f \end{aligned}$$

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Poynting – Thomson solid (1D)

$$u_{tt} = \sigma_x + f,$$

$u(t, x)$ displacement.

Constitutive relation

$$\sigma + \lambda_1 \sigma_t = \delta + \lambda_2 \delta_t$$

$\delta(t, x) := u_x(t, x)$ strain.

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Constitutive relation (c concentration)

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Poynting – Thomson solid — nonlinear model (1D)

$$u_{tt} = \chi(u_x(t))_x + \int_0^t (e^{-a(c(t,x))(t-s)}\psi(u_x(s)))_x \mathrm{d}s + f(t)$$

studied by Renardy, Hrusa and Nohel; Dafermos and Nohel and others (without chemicals)

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1 Motivation

2 Results

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$$\operatorname{div} v = 0, \quad (1)$$

$$v_t(t,x) = \Delta v(t,x) + \operatorname{div} \int_0^t a(c(t,x), t-s)\nabla v(s,x)\mathrm{d}s + f(t,x), \quad (2)$$

$$c_t(t,x) = \operatorname{div} \left((1 + |\nabla v(t,x)|^2)^{\beta/2} \nabla c(t,x) \right). \quad (3)$$

similar to Bulíček, Málek and Rajagopal

Theorem

Let $0 \le \beta < 1$, $f \in L^2([0,T], W^{-1,2}(\Omega))$, $v_0 \in L^2(\Omega)$, $c_0 \in L^2(\Omega)$. There exists a weak solution (v, c) to (1)-(3) with
$v \in L^\infty([0,T], L^2(\Omega)) \cap L^r([0,T], W_0^{1,2}(\Omega))$ with
$v_t \in L^2([0,T], W^{-1,2}(\Omega))$ and
$c \in L^\infty([0,T], L^2(\Omega)) \cap L^2([0,T], W^{1,2}(\Omega))$ with
$c_t \in L^{\frac{2r}{r+2\beta}}([0,T], W^{-1,\frac{2r}{r+2\beta}}(\Omega))$.

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$$\begin{aligned} \operatorname{div} v(t,x) &= 0 \\ v_t(t,x) &= \operatorname{div} S(c,\nabla v)(t,x) + f(t,x), \\ c_t(t,x) &= \operatorname{div} Q(c,\nabla v,\nabla c)(t,x), \end{aligned} \tag{4}$$

$$S : L^{q_1}(L^{q_1}) \times L^r(L^r) \times L^r(L^r) \to L^{r'}(L^{r'}),$$

$$Q : L^{q_1}(L^{q_1}) \times L^r(L^r) \times L^{\tilde r}(L^{\tilde r}) \times L^{q_1}(L^{q_1}) \to L^{q_2'}(L^{q_2'})$$

Q, S satisfy growth estimate, coercivity, monotonicity and are causal ($S(t)$ depends on values $v(s)$, $s \in [0,t]$ only).

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Nonlinear Poynting – Thomson with chemicals in 1D

$$u_{tt} = \chi(c(t,x),u_x)u_{xx} + \int_0^t a(c(t,x),t-s)\psi(u_x(s))_x ds + g \quad (5)$$

$$c_t = \mu c_{xx} \quad (6)$$

$$u_{tt} = \chi(t,x,u_x)u_{xx} + \int_0^t \partial_3 a(t,x,t-s)\psi(u_x(s))_x ds + g \quad \text{(IDE)}$$

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(A1) $\chi \in C_b^2$, $\psi \in C_b^3$ and $\chi, \psi' \geq c > 0$.
(A2) $a \in C_b^2(J \times \Omega \times J)$ with $a_{tt}, a_{ts} \in L^1(J^2(L^\infty(\Omega)))$,
$a_s(0,0,\cdot) \in L^2(J)$, a_t, a_s, $a_{xs}(T,\cdot,T-\cdot) \in L^1(J, L^\infty(\Omega))$.
(A3) $u_0 \in H^3(\Omega)$, $u_1 \in H^2(\Omega)$, $u_0(0) = u_0(1) = u_1(0) = u_1(1) = 0$.
(A4) $g, g_x, g_t \in C_b(J, L^2(\Omega))$, $g, g_x, g_t, g_{tt} \in L^2(J, L^2(\Omega))$.
(A5) $\chi'(u_0'(0))u_0''(0) + g(0,0) = \chi'(u_0'(1))u_0''(1) + g(1,0) = 0$.
(A6) a is of strong positive type.
positive type:

$$B(u,u) := \int_0^T u(t) \int_0^t a(t,s)u(s)\mathrm{d}s\mathrm{d}t = \langle u, Au\rangle \geq 0$$

Positive, decreasing, convex $\Rightarrow$ positive type (in convolutionary case).

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(A3) $u_0 \in H^3(\Omega)$, $u_1 \in H^2(\Omega)$, $u_0(0) = u_0(1) = u_1(0) = u_1(1) = 0$.
(A4) $g, g_x, g_t \in C_b(J, L^2(\Omega))$, $g, g_x, g_t, g_{tt} \in L^2(J, L^2(\Omega))$.
(A5) $\chi'(u_0'(0))u_0''(0) + g(0,0) = \chi'(u_0'(1))u_0''(1) + g(1,0) = 0$.
(A6) a is of strong positive type.
positive type:

$$B(u,u) := \int_0^T u(t) \int_0^t a(t,s)u(s)\mathrm{d}s\mathrm{d}t = \langle u, Au \rangle \geq 0$$

Positive, decreasing, convex $\Rightarrow$ positive type (in convolutionary case).

Nonlinear Poynting – Thomson with chemicals in 1D

(A1) $\chi \in C_b^2$, $\psi \in C_b^3$ and $\chi, \psi' \geq c > 0$.

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Nonlinear Poynting – Thomson with chemicals in 1D

Theorem

Assume (A1) – (A6) hold. If u_0, u_1, g, χ and ϕ are 'small enough', then (IDE) *has a unique solution $u : \Omega \times J \to \mathbb{R}$ with*

$$u, u_x, u_t, u_{xx}, u_{tx}, u_{tt}, u_{xxx}, u_{txx}, u_{ttx}, u_{ttt} \in C_b(J, L^2(\Omega)) \cap L^2(J, L^2(\Omega)).$$

If $J = \mathbb{R}_+$ then

$$u, u_x, u_t, u_{xx}, u_{tx}, u_{tt} \to 0$$

uniformly on Ω as $t \to +\infty$.

Nonlinear Poynting – Thomson with chemicals in 1D

Let $a(t, t-s) = e^{-\lambda(t)(t-s)}$, $\lambda \geq \alpha > 0$ and λ has small derivatives (is almost constant). Then a is of strong positive type.

Theorem

Assume (A1) – (A5) hold. If c_0, u_0, u_1, g, χ and ϕ are 'small enough', then (5), (6) has a unique solution $(u, c) : \Omega \times J \to \mathbb{R}$ with

$$u, u_x, u_t, u_{xx}, u_{tx}, u_{tt}, u_{xxx}, u_{txx}, u_{ttx}, u_{ttt} \in C_b(J, L^2(\Omega)) \cap L^2(J, L^2(\Omega)).$$

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