University Center for Mathematical Modeling, Applied Analysis and Computational Mathematics

Semester Seminar (Juniors and PhD students), hybrid form (in person and via Zoom),
PhD students: K3, Wednesday December 7, 2022, 09:00-10:15
Junior researchers: K3, Thursday December 8, 2022, 14:00-17:40

Schedule

Time	Speaker	Title
Wednesday, Dec 7		PhD students (10+5 minutes)
Session I		Chair: Josef Málek & Karel Tůma
9:00	Alena Jarolímová	Determining the Navier slip parameter in the descending aorta using 4D PC-MRI data
9:15	Anna Doležalová	Weak limit of homeomorphisms in $W^{1,n-1}$ and the (INV) condition
9:30	Jiří Malík	Cryobot simulation - thermomechanical modeling of melting rate
9:45	Jana Brunátová	Blood flow modeling in patient-specific aneurysm geometries
10:00	Martin Sýkora	GENERIC and its Application to Mixtures
Thursday, Dec 8		Junior researchers and a student (15+5 minutes)
Session II		Chair: Josef Málek
14:00	*Opening*	Josef Málek
14:05	Marek Cúth	Pure mathematics at the MathMAC project
14:25	Stefano Pozza	A new matrix equation expression for the solution of non-autonomous linear systems of ODEs
14:45	Karel Tůma	Axial pulling of a neo-Hookean fiber embedded in a generalized neo-Hookean matrix
15:05	Bangwei She	Motion of a pendulum filled with a compressible fluid
15:25	*Break*	15 min
Session III		Chair: Vít Průša
15:40	Pei Su	Asymptotic analysis of the system describing a small moving rigid body in viscous incompressible fluid
16:00	Petr Šácha	Unravelling climate impacts of atmospheric internal gravity waves
16:20	Michal Pavelka	Smoothed particle hydrodynamics for complex flows
16:40	*Break*	15 min
Session IV		Chair: Luboš Pick
16:55	Joonas Niinikoski	On quantitative Alexandrov's theorems
17:15	Hana Turčinová	Fine properties of functions from Sobolev spaces and rearrangement-invariant spaces
17:30	*Closing discussion*	10 min

Abstracts

Jana Brunátová: Blood flow modeling in patient-specific aneurysm geometries. Abstract: Motivated by the possibility of predicting the onset of aneurysm growth and rupture, computational fluid dynamics (CFD) has recently been utilized to evaluate hemodynamics and various hemodynamic parameters. We performed a retrospective study on a few patient-specific aneurysm geometries with known rupture sites that were identified by a surgeon. We compared the stationary flow inside two aneurysms with particle image velocimetry measurements and the results were in good agreement. Furthermore, we focus on the quantification of the differences in hemodynamics caused by the change in the constitutive model (both material and interface constitutive equations). The novelty of our approach is prescribing the Navier slip BC instead of the common no-slip.

Marek Cúth: Pure mathematics at the MathMAC project. I will recall my research interests during the last 5 years during which I have been part of our research group.

Anna Doležalová: Weak limit of homeomorphisms in $W^{1,n-1}$ and the (INV) condition. Abstract: The (INV) condition of Conti and De Lellis describes a kind of injectivity property, which is not always satisfied for weak limits of homeomorphisms in $W^{1,n-1}$. Roughly speaking, it says that a ball under the deformation does not lose any material from inside of it nor does it gain new material from outside. We show sufficient assumptions when the (INV) condition holds for the limits when we assume additionally the integrability of the adjoint (i.e., regularity of the inverse mappings). The talk is based on a joint work with S. Hencl and A. Molchanova.

Ondřej Chrenko: Planet migration in dusty protoplanetary disks. I will summarize recent advancements in our study of planet migration in protoplanetary disks consisting of two components: gas and dust. Although the dust-to-gas mass ratio in protoplanetary disks is generally low, our simulations suggest that the aerodynamic back-reaction of dust grains can modify the gas flow pattern near embedded planets. Consequently, the angular momentum exchange between the planet and the gas disk becomes modified and the migration rate of the planet changes compared to models that consider purely gaseous disks. I will review preliminary results of our simulations and outline (semi)-analytical pathways that might help us to understand the physics of the gas-dust interplay.

Alena Jarolímová: Determining the Navier slip parameter in the descending aorta using 4D PC-MRI data. Abstract: We investigate a computational model which is able to use variational data assimilation to reconstruct blood flow in patient-specific geometries from 4D PC-MRI images. The model uses the Navier slip boundary condition, which is able fit the MRI data more accurately. One of the goals of the study is to develop a computational tool that could estimate the slip parameter on the aortic wall from the real patient MRI data.

Jiří Malík: Cryobot simulation - thermomechanical modeling of melting rate. In this talk, which presents results obtained in collaboration with Ondřej Souček, a multi-physics modelling of Cryobot, an autonomous ice penetration vehicle for space exploration, will be introduced. The model couples a mechanical problem, governed by the Navier-Stokes equations with temperature dependent material properties, together with a thermal problem, in the form of a non-linear heat equation reformulated in enthalpy. Preliminary results show that the coupled problem formulated in the axisymmetric two-dimensional approximation is (computationally) tractable and, at the same time, estimates the rate of ice penetration based on the external conditions, i.e., ice temperature and viscosity, gravitational acceleration, etc. The results form a foundation for possible kinematic models of ice sheet penetration that lead to estimates of mission time requirements.

Joonas Niinikoski: On quantitative Alexandrov's theorems. The Alexandrov's soap bubble theorem states that a bounded and connected C^2-regular domain in an n-dimensional Euclidean space with constant boundary mean curvature must be a ball. This theorem has a lot of quantitative generalizations which usually state that if a domain in $\mathbb{R}^n$ is sufficiently regular and has boundary mean curvature close to a constant (in some sense) then the domain is "close" to a finite union of equisized balls. I am going to discuss something about these generalizations and, in particular, a quite recent joint work with Vesa Julin, in which we prove yet another quantitative formulation for the soap bubble theorem.

Michal Pavelka: Smoothed particle hydrodynamics for complex flows. Smoothed particle hydrodynamics (SPH) is a meshless numerical method solving the Navier-Stokes equations, that is particularly useful in problems with free surfaces. I will present two new results in SPH: a symplectic scheme that minimizes numerical energy dissipation, and an extension of SPH to complex fluids and solids (using the distortion field within SPH). This is a joint work with O. Kincl (Charles U.), I. Peshkov (U. Trento), and V. Klika (Czech Technical U.).

Stefano Pozza: A new matrix equation expression for the solution of non-autonomous linear systems of ODEs. The solution of systems of non-autonomous linear ordinary differential equations is crucial in a variety of applications, such us nuclear magnetic resonance spectroscopy. A new method with spectral accuracy has been recently introduced in the scalar case. The method is based on a product that generalizes the convolution. In this work, we show that it is possible to extend the method to solve systems of non-autonomous linear ordinary differential equations (ODEs). In this new approach, the ODE solution can be expressed through a linear system that can be equivalently rewritten as a matrix equation.

Bangwei She: Motion of a pendulum filled with a compressible fluid. We study the motion of a coupled system, S, constituted by a physical pendulum with an interior cavity entirely filled with a viscous, compressible fluid. We prove that, under appropriate assumptions, the fluid acts as a damper, namely, S must eventually reach a rest-state. We show by numerical tests an interesting phenomenon that “larger” compressibility favors the damping effect.

Pei Su: Asymptotic analysis of the system describing a small moving rigid body in viscous incompressible fluid. We consider the interaction between a viscous incompressible fluid and a small rigid body, which is immersed in the fluid. The fluid is modeled by the 3D Navier-Stokes equations. The motion of the body obeys the conservation of linear and angular momentum. We assume that the density of the body is constant which is independent of the size of the body ϵ. Based on the $L^p - L^q$ estimate of the so-called fluid-structure semigroup, we obtain a uniform estimate for the velocity of the body. In particular, we show that these estimates are ϵ-invariant. This help us to construct an approximation sequence of the test functions of the fluid-body system. We prove that the so- lution of the fluid-body system, in appropriate sense, converges to a solution of the Navier-Stokes equations as $\epsilon \to 0$.
This is a joint work with Jiao He (Université de Paris-Saclay).

Martin Sýkora: GENERIC and its Application to Mixtures. Hamiltonian mechanics is known for its universal applicability. The Hamiltonian equations for a mass particle are a well known concept, but their analogy can be also used to describe the reversible part of continuum physics. When combined with gradient mechanics (which describes the irreversible part) into the so called GENERIC framework, they are an interesting alternative to the classical irreversible thermodynamics based on balance laws. Moreover, the two approaches are often compatible and the former one can thus be understood as a way to obtain constitutive relations in the latter one. In our talk, we will briefly recall the principles of GENERIC and show a concrete example of a non-standard constitutive relation that it implies in the case of mixtures.

Petr Šácha: Unravelling climate impacts of atmospheric internal gravity waves. In the atmosphere, internal gravity waves (GWs) are an ubiquitous phenomenon influencing its dynamics and composition across layers. In current generation chemistry-climate models (CCMs), GWs are usually smaller than the model resolution and their effects must be parameterized. However, many aspects of their effects remain poorly understood, constrained or are not parameterized in the models. This presentation will combine observational evidence, idealized modeling and dynamical analysis of a CCM output to study short-term and climate effects of parameterized GWs. Our results demonstrate the need for improved GW parameterizations as correct representation of GW effects is vital for reliable future climate projections.

Hana Turčinová: Fine properties of functions from Sobolev spaces and rearrangement-invariant spaces. Let $\Omega \subset \mathbb{R}^n$ be a regular domain and let $d(x) = dist(x, \partial\Omega)$. Given $p \in (1, \infty)$ and a scalar function u of several variables, we seek minimal requirements on the regularity of the function u/d in order that u belongs to the Sobolev space $W_0^{1,p}$ sheltering functions with zero boundary traces. We present a new such condition in terms of Lorentz spaces and we discuss regularity of domain Ω. We will further present another recent collection of results based on improvement of the discretization technique for weighted rearrangementinvariant norms by eliminating "non-degeneracy" restrictions on the involved weights. The original discretization and antidiscretization technique appeared in the paper Discretization and anti-discretization of rearrangement-invariant norms, by A.Gogatishvili and L.Pick (Publ. Mat. (2003)), where non-degeneracy conditions on appropriate weights were assumed. In the recent work, in the case of weights that would be otherwise excluded by the restrictions, it is shown that additional limit terms naturally appear in the characterizations of the optimal constant of embedding. In three papers, we characterized embedding of Lorentz spaces $\Gamma_u^p(v) \hookrightarrow \Lambda^q(w)$, we studied three-weight inequality for the superposition of the Hardy operator and the Copson operator and we characterized embedding between two generalized weighted Lorentz spaces of type $G\Gamma$. These problems are also motivated by properties of Sobolev functions.

Karel Tůma: Axial pulling of a neo-Hookean fiber embedded in a generalized neo-Hookean matrix. We study the mechanical behavior of a slightly compressible neo-Hookean fiber, which is subjected to an axial pullout displacement, embedded in a slightly compressible generalized neo-Hookean matrix. We study three different boundary value problems containing both fully and partially embedded fibers. We study the effect of material and geometric parameters on the force required to axially displace the fiber, the shear stress at the interface and in the interior of the fiber–matrix system, and the norm of the Green-St. Venant strain. We found an interesting result in that the maximum shear stress occurs in the interior of the matrix when the shear modulus of the fiber is comparable to that of the matrix. Furthermore, as the fiber and matrix becomes more compressible, the maximum shear stress decreases. This is a joint work with P. Kar, M. Myneni, K.R. Rajagopal and C.C. Benjamin from Texas A&M University.