

# University Center for Mathematical Modeling, Applied Analysis and Computational Mathematics

Semester Seminar, hybrid form (in person and via Zoom),  
K6, Tuesday June 7, 2022, 8:55-12:10

## SCHEDULE

Time	Speaker	Title
Morning I		Zoom; Chair: Vít Dolejší
8:55	<i>Opening</i>	
9:00	Malte Kampschulte	Hard collisions of elastic bodies in solid mechanics
9:20	Joonas Niinikoski	Flat solutions to volume preserving mean curvature flow
9:40	Pei Su	An introduction of a class of fluid-structure interaction systems
10:00	<i>Break</i>	20 min
Morning II		Zoom; Chair: Vít Průša
10:20	Ondřej Chrenko	Trapping Neptunes at the inner rim of protoplanetary disks
10:40	Michal Pavelka	Hamiltonian mechanics and gradient dynamics lead to the concept of natural configurations and vice versa
11:00	<i>Break</i>	20 min
Morning III		Zoom; Chair: Josef Málek
11:20	Marek Cúth	Characterizations of weakly K-analytic and Vařák spaces using projectional skeletons and using separable PRI
11:40	Karel Tůma	Deflation technique to find multiple local minima of non-convex functionals
12:00	<i>Closing discussion</i>	10 min

## ABSTRACTS

**Marek Cúth: Characterizations of weakly K-analytic and Vařák spaces using projectional skeletons and using separable PRI.** I will talk about decompositions of nonseparable Banach spaces into separable pieces (using systems of projections), some applications and our recent contribution. The talk is based on a joint preprint with C. Correa and J. Somaglia.

**Ondřej Chrenko: Trapping Neptunes at the inner rim of protoplanetary disks.** The occurrence rate of observed exoplanets in the mass range of sub-Neptunes and Neptunes rises steeply at orbital periods of 10 days (orbital radii of 0.1 astronomical units). It has been suggested that these planets migrated in their natal protoplanetary disk and became trapped at the inner disk rim, which is where a transition in the disk viscosity is expected to occur. We perform 2D hydrodynamic simulations using the Fargo3D code to assess if the trapping is indeed possible. Our results indicate that the trapping only takes place if a vortex forms in the corotation region of the planet and its gravitational torque prevents inward migration. Furthermore, the trapping efficiency puts valuable constraints on the level of viscosity (i.e. the turbulent stress) at the inner disk rim.

**Malte Kampschulte: Hard collisions of elastic bodies in solid mechanics.** Solid/solid collisions are commonly modelled using a phenomenological approach, e.g. by adding an artificial soft repulsion potential. While this is often beneficial from the point of calculations, it turns out to not be strictly necessary. Instead, for bulk elastic solids, the physics can be fully determined by only prescribing the non-interpenetration of matter. The resulting problem however is highly non-convex. Nevertheless, we are able to show long-time existence of solutions and can derive some of their properties. This is ongoing work with G.Gravina and A.Čeřík.

**Joonas Niinikoski: Flat solutions to volume preserving mean curvature flow.** I will discuss about a recent joint project with V. Julin. The minimizing movement scheme developed by Almgren, Taylor and Wang (and independently by Luckhaus and Sturzenhecker) provides a way to construct weak solutions (flat solution) for gradient flows. Advantages of such solutions are that they do not require any comparison principle and exist at all times.

The minimizing movement method was first implemented to the case of volume preserving mean curvature flow (VMCF) by Mugnai, Seis and Spadaro and later the asymptotic behavior of such flat solutions has been studied in low dimensions. It has still been remained unanswered if a VMCF flat solution agrees with a classical VMCF solution when an initial data is sufficiently regular.

We give a positive answer to this question. First, by using so called two point method, we show that if the initial set  $E_0$  satisfies a uniform ball condition with radius  $r$ , then every flat solution  $E_t$  starting from  $E_0$  also satisfies a uniform ball condition with radius  $r - Ct$  for a short time  $T$ , where  $T$  and  $C$  depend only on the dimension, the radius  $r$  and the initial volume  $|E|$ . This implies partial consistency with a classical VMCF solution. Second, we show that if the initial set  $E_0$  is smooth, we also control the higher order derivatives of a corresponding flat solution (or rather approximative flat solutions) for a short time. This allows us to conclude the full consistency with a classical solution.

**Michal Pavelka: Hamiltonian mechanics and gradient dynamics lead to the concept of natural configuration and vice versa.** Reversible Hamiltonian mechanics and irreversible gradient dynamics form the framework of General Equation for Non-Equilibrium Reversible-Irreversible Coupling (GENERIC). This framework describes a wide range of phenomena in physics, including kinetic theory, theory of mixtures, electrodynamics, or continuum thermodynamics. The latter theory, on the other hand, can be also constructed by splitting the evolution to a reversible part (between a natural and the current configuration) and irreversible part (between a reference and the natural configuration). How are these two approaches, GENERIC and natural configurations, related?

**Pei Su: An introduction of a class of fluid-structure interaction systems.** We consider a Newtonian incompressible two-dimensional flow modeled by the Navier-Stokes equations in an unknown domain depending on the displacement of a structure. The structure can be a linear, possibly damped, beam or wave equation which depends on some parameters involved in the equation. The fluid equations are coupled with the structure via the kinematic condition and the action-reaction principle on the interface. In this presentation, we give an overview of the corresponding results on several different structure models described by the parameters mentioned above. This would help us to understand how the properties of the structure affect the properties of the fluid-structure coupled system.

**Karel Tůma: Deflation technique to find multiple local minima of non-convex functionals.** The classical Newton method is used to find the root of the set of non-linear equations. However, if the problem has more solutions, the Newton method finds only one according to the initial guess. We present a deflation technique to improve this insufficiency of the Newton method in finding multiple solutions. The method is applied to find local minima of non-convex functionals by solving the corresponding Euler-Lagrange equations. Specifically, it is used to solve the problem of buckling, in which the elastic solid under loading has a lower energy when it is not deformed uniformly.