University Center for Mathematical Modeling, Applied Analysis and Computational Mathematics

Semester Seminar, in cyberspace via Zoom, June 15, 22 and 29, 2020, 14:00-16:10

Schedule

Time	Speaker	Title
June 15		Opening at 14:00
14:10	Sebastian Schwarzacher	A variational approach to fluid-structure interactions
14:30	Gianmarco Sperone	Solenoidal extensions in domains with obstacles: explicit bounds and applications to inflow-outflow problems
14:50	Karel Tůma	Fluid-structure interaction within the Eulerian framework: application to the billiard
15:10	Giovanni Gravina	Criteria for particle rebound in the absence of collisions
15:30	Malte Kampschulte	Collisions of bulk (visco-)elastic solids
15:50	Michal Pavelka	Dynamic Maximum Entropy reduction
June 22		Starting at 14:00
14:10	Kathryn Lund	Compress-and-restart block Krylov subspace methods for Sylvester matrix equations
14:30	Marek Cúth	The complexity of isometry classes of Banach spaces
14:50	Josef Hanuš	Shapes of large main-belt asteroids
15:10	Stefano Pozza	Decay properties and the Rational Krylov Subspace Method
15:30	Scott Congreve	Adaptive Refinement for *hp*–Version Trefftz Discontinuous Galerkin Methods for the Homogeneous Helmholtz Problem
June 29		Starting at 14:00
14:10	Jaroslav Haas	Not so general suppression of the Kozai-Lidov oscillations by spherical potential
14:30	Klára Kalousová	Enthalpy method for two-phase mixtures – Application to icy moons
14:50	Michal Bathory	Analysis of unsteady flows of incompressible heat-conducting rate-type viscoelastic fluids with stress-diffusion
15:10	Mark Dostalík	Thermodynamically consistent derivation of a micro-macro model for dilute polymeric fluids
15:30	Zdeněk Mihula	Poincaré-Sobolev inequalities on the entire space

Abstracts

Michal Bathory: Analysis of unsteady flows of incompressible heat-conducting rate-type viscoelastic fluids with stress-diffusion. In this talk we will focus on the main difficulties that arise in the existence analysis of a certain viscoelastic fluid model. In this model the basic Navier-Stokes-Fourier system is coupled to a (tensorial) equation describing evolution of the "elastic part" of the fluid. Due to this coupling the standard methods known for NSF systems may not apply. Nevertheless, we can still show that under reasonable assumptions there exists a weak solution fulfilling basic physical requirements.

Scott Congreve: Adaptive Refinement for *hp*–Version Trefftz Discontinuous Galerkin Methods for the Homogeneous Helmholtz Problem. We consider a Trefftz discontinuous Galerkin finite element (TDG) approximation of the solution to the homogeneous Helmholtz problem $-\Delta u - k^2 u = 0$ in $\Omega \subset \mathbb{R}^d$, $d = 2, 3$, for a given fixed wavenumber k. The TDG method for Helmholtz is a discontinuous finite element methods, where instead of using standard polynomial basis functions on each element we instead use (local) solutions to the Helmholtz equation. Here, we consider use of plane wave functions $\mathrm{e}^{ik\boldsymbol{d}_\ell\cdot(\boldsymbol{x}-\boldsymbol{x}_K)}$, where $\boldsymbol{d}_\ell$, $\ell = 1, \ldots, p_K$, are distinct unit propagation directions on an element K with element centre $\boldsymbol{x}_K$. As the chosen direction are distinct, this leads to a finite element space on each element with dimension p_K. We also consider the so-called *effective polynomial degree* q_K of an element, from which we can derive the number of plane waves, such that the convergence rate of the TDG formulation is the same as the convergence rate of the corresponding polynomial basis function finite element approximation with polynomial degree q_K. It has been shown that p_K is of order q_K^{d-1}, compared to the number of degrees of freedom for the polynomial-based approximation which is of order q_K^d.

Existing work has already studied adaptive h-refinement of the finite element mesh (refinement of mesh element size), and we present a basic, numerically derived, extension to this work to handle p-refinement (increasing the number of plane wave basis functions per element). We also present a modified version of an existing algorithm for deciding when to perform h- or p-refinement.

We also consider the fact that wave propagation problems often have a primary propagation direction and aligning one of the directions of the plane wave basis functions with this primary direction can reduce the error. We show how we can use the eigenvalues and eigenvectors of the Hessian of the numerical solution to the Trefftz discontinuous Galerkin finite element method to approximate this primary propagation direction and, hence, adaptively adjust the selected plane wave directions for the plane wave basis functions.

Marek Cúth: The complexity of isometry classes of Banach spaces. In my talk I will present our recent paper "The complexity of isometry classes of Banach spaces", where we investigate how difficult it is to classify various separable Banach spaces (e.g. c_0 or ℓ_p and L_p for $p \in [1, \infty)$) up to isometry. We develop a very precise tool which enables us to compare the complexity of various classification problems and, using this tool, we prove e.g. that ℓ_2 is the only one separable infinite-dimensional Banach space admitting the easiest possible characterization up to isometry and up to isomorphism.

Mark Dostalík: Thermodynamically consistent derivation of a micro-macro model for dilute polymeric fluids. The rheology of complex fluids such as polymeric liquids is highly non-Newtonian in nature and manifests itself as an extra stress component in the Cauchy stress tensor. At the purely macroscopic level, the extra stress tensor is linked to the velocity field through, say, a partial differential equation. An alternative approach consists in finding an expression for the macroscopic extra stress tensor in terms of the microscopic dynamics of the polymer chains. We present a thermodynamically based approach to the design of a class of such micro-macro models for dilute polymeric liquids and show that the thermodynamical background of the model naturally yields stability of the steady state when the fluid occupies an isolated vessel.

Giovanni Gravina: Criteria for particle rebound in the absence of collisions. In this talk, we will discuss some analytical conditions for the rebound of an elastic body moving towards a wall in a viscous incompressible fluid. We will focus on the delicate case of no-slip boundary conditions, where fascinating phenomena can be observed due to the fact that particle-wall collisions cannot occur in finite time.

Jaroslav Haas: Not so general suppression of the Kozai-Lidov oscillations by spherical potential. A Keplerian orbit of two gravitationally bound bodies that are exposed to the influence of a third, distant body may undergo severe changes of its shape and orientation, the so-called Kozai-Lidov oscillations of orbital elements. It has become a traditional statement that these are suppressed if such a system is embedded in an additional spherically symmetric gravitational potential. Even though this is true in a statistical sense, astrophysically relevant exceptions do exist. While for some of them, the suppression is just not that effective, in other cases, the oscillations are even enhanced.

Josef Hanuš: Shapes of large main-belt asteroids. The recent images of the largest main belt asteroids unveiled strikingly different worlds, which illustrates the complex compositional and geological diversity of this population of objects. In this talk, I will show the high angular resolution observations of several large main belt asteroids performed with the powerful Adaptive-Optics (AO) SPHERE imiger mounted on the Very Large Telescope (VLT). These observations related to ESO's large observing campaign were obtained within an international collaboration effort. I will focus on differences between shapes of observed asteroids, especially with respect to their sizes, multiplicity and bulk composition.

Klára Kalousová: Enthalpy method for two-phase mixtures – Application to icy moons. Outer solar system moons have thick ice layers where melting may occur. We investigated the melting process and the subsequent meltwater transport through a convecting solid ice using a two-phase one-component mixture model. The system of governing equations comprised the Stokes system, the Darcy equation and two evolution equations – for temperature and porosity (water content). While relatively straightforward, such approach brings implementation and numerical problems. The enthalpy method allows to solve only one evolution equation for the mixture enthalpy and to evaluate temperature and porosity by a straightforward postprocessing procedure. This is a joint project with Ondrej Soucek.

Malte Kampschulte: Collisions of bulk (visco-)elastic solids. While elastic collisions are studied at all levels, from high-school to research level, there are always additional assumptions used, such as energy conservation. In this talk we examine why these assumptions are often necessary, but show that in the right model one can do without. In particular we give some preliminary existence results for bulk solids, where it turns out that simple non-interpenetration of matter in the style of an obstacle problem is the only condition required.

Kathryn Lund: Compress-and-restart block Krylov subspace methods for Sylvester matrix equations. Block Krylov subspaces comprise building blocks in many state-of-the-art solvers for large-scale matrix equations as they arise, e.g., from the discretization of partial differential equations. Extended block Krylov subspace methods (KSMs) provide a major reduction in iteration counts over polynomial block KSMs, but they require reliable solvers for the coefficient matrices, and these solvers are often iterative methods themselves. It is not hard to devise scenarios in which the available memory, and consequently the size of the Krylov basis, is limited. In such scenarios for linear systems, restarting is a well explored technique for mitigating memory constraints while still decreasing the residual. In this work, such restarting techniques are applied to polynomial KSMs for matrix equations with a compression step to control the growing rank of the residual. An error analysis is also performed, leading to heuristics for dynamically adjusting the basis size in each restart cycle. A panel of numerical experiments demonstrates the effectiveness of the new method with respect to extended block KSMs.

Zdeněk Mihula: Poincaré-Sobolev inequalities on the entire space. Abstract: The aim of this talk is to characterize the validity of the inequality $\|u\|_{Y(R^n)} \leq C\|\nabla^m u\|_{X(R^n)}$, where X and Y are rearrangement-invariant spaces on the entire R^n and u is a m–times weakly differentiable function whose m-th gradient $\nabla^m u$ is in X and that satisfies a certain mild assumption on its growth at infinity. If we impose no restriction on the growth of Sobolev functions at infinity, such an inequality cannot hold. Nevertheless, we also characterize the inequality $\|u - P(u)\|_{Y(R^n)} \leq C\|\nabla^m u\|_{X(R^n)}$, where $P(u)$ is a certain polynomial depending on the function u and the function is, unlike in the former inequality, subject to no restriction on its growth at infinity.

Mihal Pavelka: Dynamic Maximum Entropy reduction. Consider some detailed dynamics like the Maxwell model for non-Newtonian fluids. Does such detailed dynamics correspond at least approximatevely to solutions to simpler dynamics like Navier-Stokes equations? We provide a feasible heuristical method called the Dynamic MaxEnt, by which the reduction of the detailed dynamics to simpler equations is carried out. Another examples are the reduction of hyperbolic heat conduction to the Fourier law or the reduction of hyperbolic mass transport to the non-isothermal Maxwell-Stefan equations. A particular observation is that parity (with respect to time-reversal) of the quantities being reduced changes by the reduction.

Sebastian Schwarzacher: A variational approach to fluid-structure interactions. I introduce a recent developed variational approach for hyperbolic PDE's. The method allows to show the existence of weak solutions to fluid-structure interactions where a visco-elastic bulk solid is interacting with an incompressible fluid governed by the unsteady Navier Stokes equations. This is a joint work with M. Kampschulte and B. Benesova. I will also provide some outlook on future works.

Stefano Pozza: Decay properties and the Rational Krylov Subspace Method. Given a banded matrix A, the matrix function $f(A)$ is generally a full matrix. Nonetheless, the magnitude of $f(A)$ elements typically decays with the distance from the band, allowing to approximate $f(A)$ as a banded matrix. Krylov subspace methods are based on a model reduction strategy in which the input matrix A is transformed into T, a smaller banded matrix. A-priori bounds for the decay behavior of the elements in $f(T)$ has been successfully used to improve Krylov subspace methods. The rational Krylov subspace method is also based on the reduction of the input matrix A into a smaller matrix J. However, in this case, J is generally full. We will present a-priori decay bounds, which show that J exhibits a decay phenomenon nevertheless.

Gianmarco Sperone: Solenoidal extensions in domains with obstacles: explicit bounds and applications to inflow-outflow problems. We introduce a new method for constructing solenoidal extensions of fairly general boundary data in (2D or 3D) cubes that contain an obstacle; this method allows us in particular to provide explicit bounds for the gradient of the extensions. It runs as follows: by inverting the trace operator, we first determine suitable extensions, not necessarily solenoidal, of the data; then we solve the Bogovskii problem with the resulting divergence to obtain a solenoidal extension; finally, by solving a variational problem involving the infinity-Laplacian and using ad hoc cutoff functions, we find explicit bounds in terms of the geometric parameters of the obstacle. The natural applications of our results lie in the analysis of inflow-outflow problems, in which an explicit bound on the inflow velocity is needed in order to estimate the threshold for bifurcation from uniqueness in stationary Navier-Stokes equations modelling a flow around an obstacle. This is a joint work with Ilaria Fragalà and Filippo Gazzola (Politecnico di Milano, Italy).

Karel Tůma: Fluid-structure interaction within the Eulerian framework: application to the billiard. In this talk, we present a fully Eulerian model for fluid-structure interaction capable of describing the motion of elastic solid inside the fluid. Such problem can not be modeled using the arbitrary Lagrangian-Eulerian (ALE) method on a fixed mesh since the Jacobian of the ALE mapping becomes singular during the deformation. Here, we use a unifying setting for solid and fluid and the constituents are distinguished by the level-set parameter that changes smoothly across the interface. The model is implemented in the finite element method and applied to the problem when at t=0 the elastic ball is put into the container filled with the fluid at rest. Initially, we prescribe a non-zero ball velocity and simulate how the ball bounces from the edge of the container with the no-slip boundary condition. We study how the viscosity of the fluid influences the solution of the problem.