

# University Center for Mathematical Modeling, Applied Analysis and Computational Mathematics

Semester Seminar, 21<sup>th</sup> May 2018, 8:45–12:20, Room K4

## SCHEDULE

Time	Speaker	Title
8:45		Opening
8:45	Marek Cúth	Complexity of distances between metric and Banach spaces
9:05	Jaroslav Haas	Kozai-Lidov dynamics in galactic nuclei
9:25	Josef Hanuš	Thermophysical modeling of main-belt asteroids from WISE thermal data – the VS-TPM method
9:45		Coffee Break
9:55	Ondřej Souček	Viscous deformation of the ice crust of Enceladus on a geological time scale – implications for the heat flux
10:15	Klára Kalousová	Transport processes within the high-pressure ice layers of Ganymede and Titan – implications for volatiles transport
10:35	Michal Pavelka	Multiscale thermo-dynamics of radiation
10:55		Coffee Break
11:05	Karel Tůma	Solving viscoelastic rate-type fluid models like Oldroyd-B or Burgers for the price of Navier-Stokes
11:25	Miloslav Vlasák	A priori error estimates for space-time discontinuous Galerkin method combined with Proper orthogonal decomposition
11:45	Stefano Pozza	Gauss quadrature for quasi-definite linear functionals
12:05		Conclusion

## ABSTRACTS

**Marek Cúth: Complexity of distances between metric and Banach spaces.** I will talk about our recent work in progress with M. Doucha and O. Kurka, where we investigate how difficult it is to classify metric/Banach spaces up to small distance in a certain sense (the distance is zero if the spaces are isometric, so the small distance expresses the fact that the spaces are close to being isometric). We consider various notions of distances such as Gromov-Hausdorff, Kadets, and Banach-Mazur distances. I will try to give the audience certain insight to this area of mathematics called “Invariant descriptive theory” and briefly mention some of our results.

**Jaroslav Haas: Kozai-Lidov dynamics in galactic nuclei.** Various approximations of the gravitational 3-body problem have been investigated throughout the history. One of them, the so-called hierarchical 3-body problem, in which the third body orbits the inner binary on a much wider orbit, has proved itself a very handy tool for description of various dynamical effects in the Solar system. The recently obtained results of N-body modelling of the Galactic Centre suggest, however, that this classical problem pioneered by Y. Kozai and M. L. Lidov plays an important role even in the evolution of complex N-body systems containing thousands to millions of bodies. In my talk, I will describe some of its possible consequences.

**Josef Hanuš: Thermophysical modeling of main-belt asteroids from WISE thermal data – the VS-TPM method.** By means of a varied-shape thermophysical model (VS-TPM) of Hanuš et al. (2015, *Icarus*, 256) that takes into account asteroid shape and pole uncertainties, we analyze the thermal infrared data acquired by the NASA’s Wide-field Infrared Survey Explorer of about 300 asteroids. We utilize their publicly available convex shape models and rotation states as input for the thermophysical modeling. For more than one hundred asteroids, the thermophysical modeling gives us an acceptable fit to the thermal infrared data allowing us to report their thermophysical properties such as size, thermal inertia, surface roughness or visible geometric albedo. This work more than doubles the number of asteroids with determined thermophysical properties, especially the thermal inertia. Finally, we present the main results of the statistical study of derived thermophysical parameters within the whole population of main-belt asteroids and within few asteroid families. The thermal inertia increases with decreasing size, but a large range of thermal inertia values is observed within the similar size ranges between  $D \sim 10 - 100$  km. We derived unexpectedly low thermal inertias ( $< 20 \text{ J m}^{-2}\text{s}^{-1/2}\text{K}^{-1}$ ) for several asteroids with sizes  $10 < D < 50$  km, indicating a very fine and mature regolith on these small bodies.

My talk will highlight the main results (see above) of our recent paper published in *Icarus* (Hanus et al. 2018) with the emphases on the technical details of the VS-TPM method.

**Ondřej Souček: Viscous deformation of the ice crust of Enceladus on a geological time scale – implications for the heat flux.** We present a novel extension of our 3D finite element code for solving the deformation of Enceladus's ice shell on geological time scales, describing the ice as an incompressible highly viscous Newtonian fluid. We consider realistic ice shell geometry based on the recent topography, gravity and libration inversion and a realistic viscosity contrast across the shell spanning 8 orders of magnitude. Assuming the shape of the shell to be stationary, the computed map of the normal velocity at the base of the shell allows us to infer the basal jump in the normal heat flux from the Stefan condition. The sign of the geothermal heat flux from the ocean provides a surprisingly strong constraint on the amplitude of the basal ice viscosity, favoring values at the upper bound of the existing estimates.

**Klára Kalousová: Transport processes within the high-pressure ice layers of Ganymede and Titan – implications for volatiles transport.** Ganymede and Titan, the largest icy moons in the solar system, harbor deep liquid water oceans separated from their silicate interiors by layers of high-pressure (HP) ice. Both moons are similar in mass and size but their radial mass distribution is different. Moreover, Titan is the only known moon with a dense, methane-rich atmosphere. Since photochemistry would remove the present-day amount of methane in only a few millions of years, its presence indicates that methane must be resupplied from within Titan and that some exchange processes may be ongoing in Titan's interior. Here, we investigate the transport processes through the HP ice layers and compare the results obtained for both moons.

**Michal Pavelka: Multiscale thermo-dynamics of radiation.** Energy transfer due to electromagnetic radiation can be studied from the perspective of different levels of description. The most detailed description is achieved within kinetic theory of photons, where the evolution equation is similar to the Boltzmann equation. A less detailed hydrodynamic description, where momentum and entropy densities play the role of state variables, can be obtained by thermodynamic reduction from the kinetic level. Finally, thermodynamic equilibrium represented by the Planck law can be recovered by further reduction. The goal is to understand the passages between those levels of description and their relations with the standard radiation models.

**Karel Tůma: Solving viscoelastic rate-type fluid models like Oldroyd-B or Burgers for the price of Navier-Stokes.** Rate-type fluid models are popular for their ability to describe many materials of completely different origin, like asphalt or vitreous body in the eye. In these models, which are convenient for describing the viscoelastic behavior, the part of the Cauchy stress tensor satisfies an evolution equation that needs to be solved. This makes the 3D simulations of these models much more expensive than with the standard Navier-Stokes model: 4 global unknowns for Navier-Stokes compared to 16 unknowns for Burgers. We present a way how to solve the governing equations for this class of models almost for the price of Navier-Stokes under the assumption that the convective term in the transport equation for the part of the stress is negligible.

**Miloslav Vlasák: A priori error estimates for space-time discontinuous Galerkin method combined with Proper orthogonal decomposition.** We assume a simple parabolic problem discretized by space-time discontinuous Galerkin method. Such a problem can be represented by a linear systems of very high order. We employ the order reduction technique Proper orthogonal decomposition that is based on finding a small suitable set of orthogonal basis functions in such a way that the maximum of the information is preserved from the original discrete solution on the given number of basis functions. We will show that the error of the reduced order problem corresponds to the classical error component and to the sum of remaining singular values coming from correlation matrix given by the original discrete solution.

**Stefano Pozza: Gauss quadrature for quasi-definite linear functionals.** The Gauss quadrature can be naturally generalized to approximate quasi-definite linear functionals where the interconnections with (formal) orthogonal polynomials, (complex) Jacobi matrices and Lanczos algorithm are analogous to those in the positive definite case. In particular, the existence of the  $n$ -weight (complex) Gauss quadrature corresponds to successfully performing the first  $n$  steps of the Lanczos algorithm.